D.R.G.Government Degree College

Tadepalligudem, West Godavari District

CBCS 2020-21 REVISED SYLLABUS

II Semester

Physics Paper-II

WAVE OPTICS

Study Material

(English Medium)

Prepared by

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B Sc	Semester: 2	Credits: 4
Course: 2	Wave Optics	Hrs/Wk: 4

Student able to Learning:

- Understand the nature of light and principles of Laser and holography.
- Analyse the intensity variation of light due to interference, diffraction and polarization.
- Solve problems in Optics by selecting the appropriate equations and performing numerical or analytical calculations.
- Student can able to operation of optical devices including polarizers, interferometers,and Lasers.

UNIT I: Interference of light: (12hrs)

Introduction, Conditions for interference of light, Interference of light by division of wave front and amplitude,Phase change on reflection- Stokes' treatment, Lloyd's single mirror, Interference in thin films: Plane parallel and wedge- shaped films, colours in thin films, Newton's rings in reflected light-Theory and experiment, Determination of wavelength of monochromatic light, Michelson interferometer anddetermination of wavelength.

UNIT II: Diffraction of light:(12hrs)

Introduction, Types of diffraction: Fresnel and Fraunhoffer diffractions, Distinction between Fresnel and Fraunhoffer diffraction,Fraunhoffer diffraction at a single slit, Plane diffraction grating,Determination of wavelength of light using diffraction grating, Resolving power of grating, Fresnel's half period zones, Explanation of rectilinear propagation of light, Zone plate, comparison of zone plate with convex lens.

UNIT III: Polarisation of light:(12hrs)

Polarized light: Methods of production of plane polarized light, Double refraction, Brewster's law, Malus law, Nicol prism, Nicol prism as polarizer and analyzer, Quarter wave plate, Half wave plate, Plane, Circularly and Elliptically polarized light-Production and detection, Optical activity, Laurent's half shade polarimeter: determination of specific rotation.

UNIT IV: Aberrations and Fibre Optics: (12hrs)

Monochromatic aberrations, Spherical aberration, Methods of minimizing spherical aberration, Coma, Astigmatism and Curvature of field, Distortion; Chromatic aberration-the achromatic doublet; Achromatism for two lenses (i) in contact and (ii) separated by a distance. **Fibre optics:** Introduction to Fibers, different types of fibers, rays and modes in an optical fiber, Principles of fiber communication (qualitative treatment only), Advantages of fiber optic communication.

UNIT V: Lasers and Holography:(12hrs)

Lasers: Introduction, Spontaneous emission, stimulated emission, Population Inversion, Laser principle, Einstein coefficients, Types of lasers-He-Ne laser, Ruby laser, Applications of lasers; Holography: Basic principle of holography, Applications of holography

INTERFERENCE

Coherent Sources of Light

If the phase difference between the light waves emitted from two sources is either zero or constant, then such sources are called coherent sources of light. The two light sources must be coherent to produce interference.

Conditions for sustained interference

The following conditions must be satisfied to produce sustained interference.

1.Coherent Sources of light:

The two light sources must be coherent to produce a sustained interference pattern. The positions of maxima and minima change continuously if the phase difference is not constant. Hence the interference pattern will not be stable.

2. Equal Frequencies or Wavelengths:

The two light sources must emit waves of same frequency or wavelength to produce a sustained interference pattern. The positions of maxima and minima change continuously if the frequencies are not equal.

3. Equal amplitudes:

The two light sources must emit waves of same amplitude to produce a clear interference pattern. Contrast between the bright and dark fringes will be maximum if the amplitudes are equal. Hence the interference pattern will be clear.

4. Separation between the coherent sources:

The separation between the two coherent sources $2d$ must be very small.

5. Distance between the source and the screen:

The distance between the source and the screen D must be large.

6. Narrow Sources of light:

The two light sources must be extremely narrow.

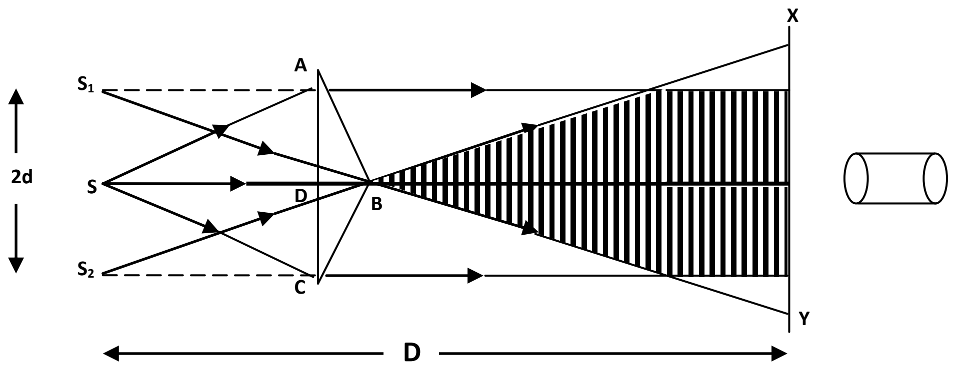
Fresnel's Biprism

Fresnel used a biprism to produce interference pattern. Biprism is a combination of two small angled prisms joined together along their bases.

Fresnel's biprism produces two virtual images of the slit. These two virtual images act as two coherent sources and hence produce interference pattern.

- S is a monochromatic source of light.
- ABCD is the Fresnel's biprism.
- Consider a screen XY placed at a distance D from the source.

- When the light emitted from the source S falls on the biprism, it is divided in to two wave fronts.
- Light rays which are incident on the upper part ABD of the biprism are bent downwards and appear to diverge from the virtual source S_1
- Similarly the light rays which are incident on the lower part BCD of the biprism are bent upwards and appear to diverge from the virtual source S_2.
- Since S_1, S_2 are the virtual images of the source S, they act as two coherent sources. The separation between these coherent sources is also very small. Hence a sustained interference pattern is produced.



Determination of wavelength of light using Fresnel's biprism

$$\text{Wavelength } \boldsymbol{\lambda} = \frac{\boldsymbol{\beta}.\mathbf{2}\boldsymbol{d}}{\boldsymbol{D}}$$

Here $\boldsymbol{\beta}$ is the fringe width, $\boldsymbol{D}$ is the distance of the screen from the source, $\mathbf{2}\boldsymbol{d}$ is the separation between the two coherent sources.

Determination of fringe width ($\boldsymbol{\beta}$):

Vertical cross wire of the eye piece of the microscope is focussed on a bright fringe and the corresponding reading x_1 is noted. Now the cross wire of the eye-piece is moved to the 20^{th} bright fringe and the corresponding reading x_2 is noted.

$$\text{Fringe width } \beta = \frac{(x_2 - x_1)}{20}$$

Determination of separation between the two sources $\mathbf{2}\boldsymbol{d}$:

To determine the value of $\mathbf{2}\boldsymbol{d}$, a convex lens with focal length less than one fourth of the distance between the eye-piece and the biprism is taken. The lens is gradually moved towards the eye-piece and the two positions of the lens L_1, $L_2$where the two images S_1, S_2 are clearly seen are identified. Let d_1,d_2 be the distances between the two images in these positions, then

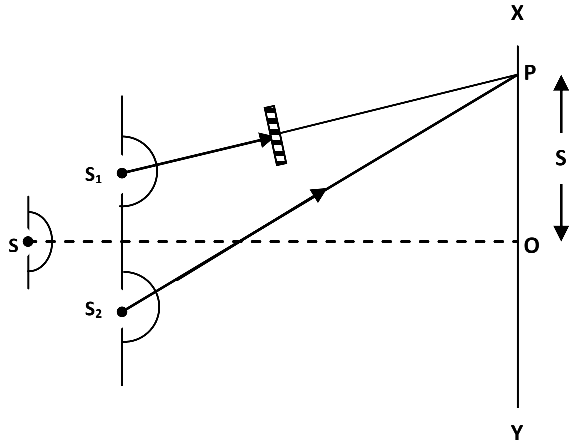
$$\frac{d_1}{2d} = \frac{v}{u} \text{ and } \frac{d_2}{2d} = \frac{u}{v}$$

$$2d = \sqrt{(d_1 d_2)}$$

The distance between the source and the eye-piece D can be measured with the help of a scale on the optical bench.

Determination of thickness of a thin transparent material

Biprism can be used to determine the thickness of a thin transparent material.



S_1, S_2 are two coherent sources of light. Consider a screen XY at a distance D from the two sources. Let a transparent sheet of thickness t and refractive index μ is placed in the path of the light waves emitted from S_1. As a result, the fringes are displaced through a certain distance. Let S be the fringe displacement. Light emitted from source S_1 travels a distance of (S_1P-t) through air and a distance of t through the transparent sheet.

The time taken by the light rays to reach the point P from S_1.

$$= \frac{S_1P - t}{C} + \frac{t\,\mu}{C} = \frac{S_1P + (\mu - 1)t}{C}$$

Similarly, the time taken by the light rays to reach the point P from S_2

$$= \frac{S_2P}{C}$$

$$\text{Path difference} = S_2P - [S_1P + (\mu - 1)t] = S_2P - S_1P - (\mu - 1)t$$

$$= \frac{2xd}{D} - (\mu - 1)t$$

For n^{th} maximum, $\frac{2x_n d}{D} - (\mu - 1)t = n\lambda$

$$x_n = \frac{D}{2d}[n\lambda + (\mu - 1)t]$$

In the absence of transparent sheet, $t = 0$.

Hence n^{th} maximum $= \frac{Dn\lambda}{2d}$

Fringe displacement $S = \frac{D}{2d}[n\lambda + (\mu - 1)t] - \frac{D\,n\lambda}{2d}$

$$\boldsymbol{S = \frac{D}{2d}(\mu - 1)t}$$

Thickness of transparent sheet $\boldsymbol{t = \frac{S\,.2d}{D(\mu - 1)}}$

Colours of thin films

We know that beautiful colours can be observed when white falls on thin films like soap bubble, oil on the surface of water etc.. This phenomenon is due to the interference of light. The light rays reflected from the top and bottom layers of the film interfere with each other to produce different colours.

Bright or dark appearance of the reflected light from a thin film depends on the path difference between the light rays reflected from the top and bottom layers of the film. It depends on the values of μ,t,r. White light has many colours. The value of refractive index μ is different for different colours. Hence only some colours satisfy the condition for maximum intensity. Only those colours will be visible in the reflected light. Remaining colours will be absent. When the film is observed with eye in different positions, the value of r is different. Hence a different set of colours is observed.

Non-Reflecting films

Lenses used in telescopes and microscopes are coated with a thin film to minimize the loss of light due to reflection. This film is called a non-reflecting film. This process is called Blooming.

Necessity of Non-reflecting film:

Consider a light ray passing through a medium of refractive index μ_1 incident normally on a medium of refractive index μ_2.

If I is the intensity of incident light, then the intensities of reflected light and transmitted light are given by

$$I_r = \left(\frac{\mu_2 - \mu_1}{\mu_2 + \mu_1}\right)^2 I$$

$$I_t = \left(\frac{2\mu_1}{\mu_2 + \mu_1}\right)^2 I$$

At air-glass boundary, $I_r = 0.04\, I$

Hence 4 % of incident light is reflected and 96 % is transmitted at the boundary of air, glass media.

The reflected light is not useful in the formation of the image. The intensity of reflected light increases when more number of lenses are used in the optical instrument. Hence the intensity of the image decreases. A non-reflecting film is coated on the lenses to minimize this loss of reflection.

Thickness of non-reflecting film:

Consider a non-reflecting film of thickness t and refractive index μ. Let a monochromatic light of wavelength λ is incident on the film. In the non-reflecting film, the light rays reflected from the top and bottom layers of the film interfere destructively.

Path difference $\Delta = 2\,\mu\, t$

For destructive interference $\Delta = (2n+1)\lambda/2$

For n=0, $\Delta = \lambda/2$

$$Hence\ 2\,\mu\, t = \lambda/2$$

$$\therefore \boldsymbol{t} = \frac{\boldsymbol{\lambda}}{\boldsymbol{4\mu}}$$

Refractive index of non-reflecting film:

For complete destructive interference, amplitudes of the two interfering waves must be equal.

$$\left(\frac{\mu - \mu_1}{\mu + \mu_1}\right)^2 I = \left(\frac{\mu_2 - \mu}{\mu_2 + \mu}\right)^2 I$$

Here μ_1 is the refractive index of the medium outside the film; μ is the refractive index of the film and μ_2 is the refractive index of the lens.

Since the medium outside the film is air, $\mu_1 = 1$

$$\left(\frac{\mu - 1}{\mu + 1}\right)^2 I = \left(\frac{\mu_2 - \mu}{\mu_2 + \mu}\right)^2 I$$

$$\left(\frac{\mu - 1}{\mu + 1}\right) = \left(\frac{\mu_2 - \mu}{\mu_2 + \mu}\right)$$

$$\frac{\mu}{\mu + 1} = \frac{\mu_2}{\mu_2 + \mu}$$

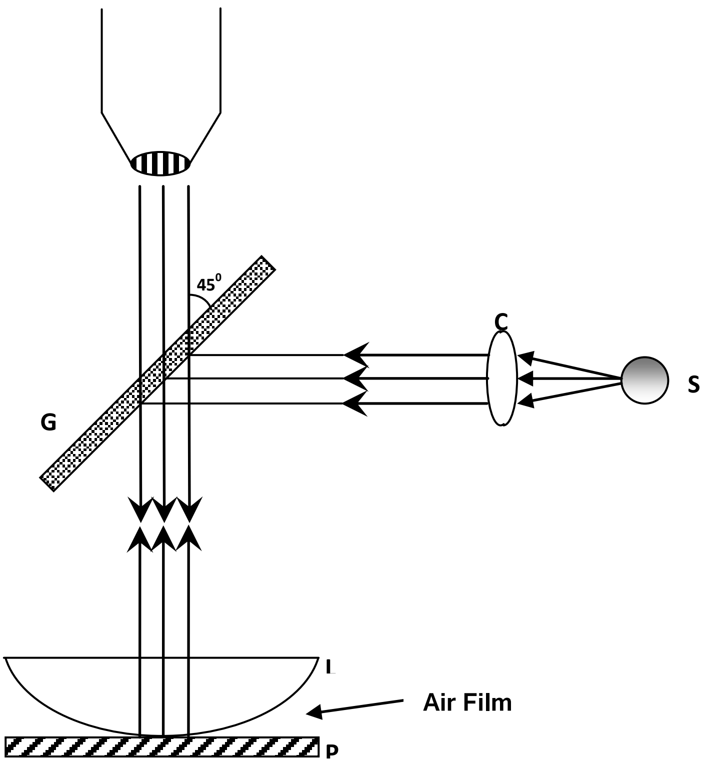
$$\mu^2 = \mu_2$$

$$\mu = \sqrt{\mu_2}$$

Hence the refractive index of the film should be equal to the square root of the refractive index of the lens.

Newton's Rings

Consider a Plano-convex lens placed on a glass plate such that the convex surface of the lens touches the glass plate. Then an air film is formed between the convex lens and the glass plate. If a monochromatic light is incident on the air film normally, then alternate bright and dark concentric circular rings are produced due to interference. These rings are called Newton's rings.



Experimental arrangement:

- Let a Plano-convex lens **L** be placed on a glass plate **P** such that the convex surface of the lens touches the glass plate.
- S is a monochromatic source of light.

- Light emitted from the source S is converted in to a parallel beam using a convex lens C.
- This parallel beam of light falls on a semi-silvered glass plate G which is inclined at an angle of 45^o with the horizontal.
- Light reflected from this glass plate G falls on the air film between the convex lens and the glass plate G normally.
- Light reflected from the air film can be observed through the microscope M.

Reason for the formation of Newton's Rings:

- ❖ Newton's rings are formed due to the interference between the light rays reflected from the top and bottom surfaces of the air film.

Path difference between the two rays

$$\Delta = 2\mu t\cos r + \lambda/2$$

For air film μ=1 and for normal incidence r=0.

$$\Delta = 2t + \lambda/2$$

At the point of contact of the glass plate and the convex lens t=0

$$\Delta = \lambda/2$$

This is the condition for minimum intensity. Hence a dark spot is produced at the centre O.

For constructive interference

$$\Delta = n\lambda$$

$$2t + \frac{\lambda}{2} = n\lambda$$

$$\mathbf{2t = \frac{(2n-1)\lambda}{2}}$$

$$n = 1,2,3 \ldots \ldots$$

For destructive interference

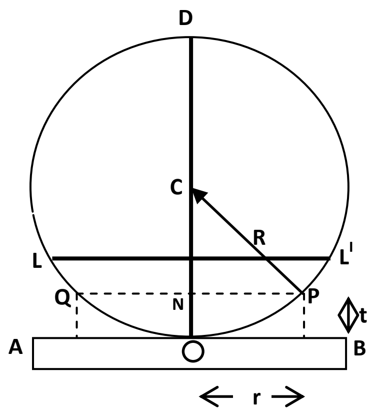
$$\Delta = (2n+1)\lambda/2$$

$$2t + \frac{\lambda}{2} = (2n+1)\lambda/2$$

$$\mathbf{2t = n\lambda}$$

$$n = 0,1,2,3 \ldots \ldots$$

<u>**Radii of bright and dark rings:**</u>



Let R be the radius of curvature of the convex lens and the λ be the wavelength of monochromatic light.

$$NP \times NQ = NO \times OD$$
$$r \times r = t \times (2R - t) = 2Rt - t^2 \approx 2Rt$$
$$r^2 = 2Rt$$
$$t = \frac{r^2}{2R}$$
$$2t = \frac{(2n-1)\lambda}{2}$$
$$2\frac{r^2}{2R} = \frac{(2n-1)\lambda}{2}$$
$$r^2 = \frac{(2n-1)\lambda R}{2}$$
$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$
$$D = \sqrt{2\lambda R}\sqrt{(2n-1)}$$
$$\boldsymbol{D \propto \sqrt{(2n-1)}}$$

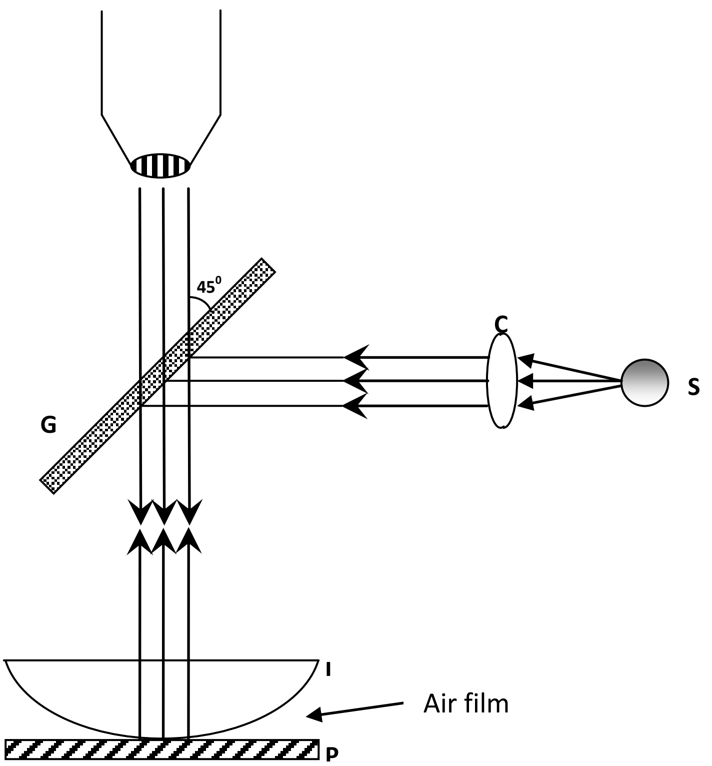
✓ Hence the diameters of bright rings are proportional to the square roots of odd natural numbers,

For dark rings

$$2t = n\lambda$$
$$2\frac{r^2}{2R} = n\lambda$$
$$r^2 = n\lambda R$$
$$\frac{D^2}{4} = n\lambda R$$
$$D = \sqrt{4n\lambda R} = 2\sqrt{\lambda R}\sqrt{n}$$
$$\boldsymbol{D \propto \sqrt{n}}$$

✓ Hence the diameters of dark rings are proportional to the square roots of natural numbers.

Determination of wavelength of light using Newton's rings



Consider a Plano-convex lens placed on a glass plate such that the convex surface of the lens touches the glass plate. Then an air film is formed between the convex lens and the glass plate. If a monochromatic light is incident on the air film normally, then alternate bright and dark concentric circular rings are produced due to interference. These rings are called Newton's rings.

Experimental arrangement:

- Let a Plano-convex lens **L** be placed on a glass plate **P** such that the convex surface of the lens touches the glass plate.
- S is a monochromatic source of light.
- Light emitted from the source S is converted in to a parallel beam using a convex lens C.
- This parallel beam of light falls on a semi-silvered glass plate G which is inclined at an angle of 45^o with the horizontal.
- Light reflected from this glass plate G falls on the air film between the convex lens and the glass plate G normally.
- Light reflected from the air film can be observed through the microscope M.

Reason for the formation of Newton's Rings:

- ✓ Newton's rings are formed due to the interference between the light rays reflected from the top and bottom surfaces of the air film.

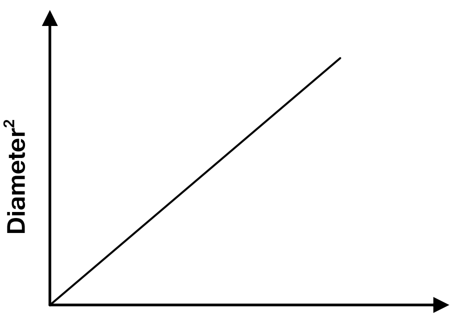
Let R be the radius of curvature of the convex lens and λ be the wavelength of monochromatic light.
Let r_n,r_m be the radii and D_n,D_m be the diameters of n^{th}, m^{th} dark rings.

$$D_n^2 = 4n\lambda R, \ D_m^2 = 4m\lambda R$$

$$D_m^2 - D_n^2 = 4(m-n)\lambda R$$

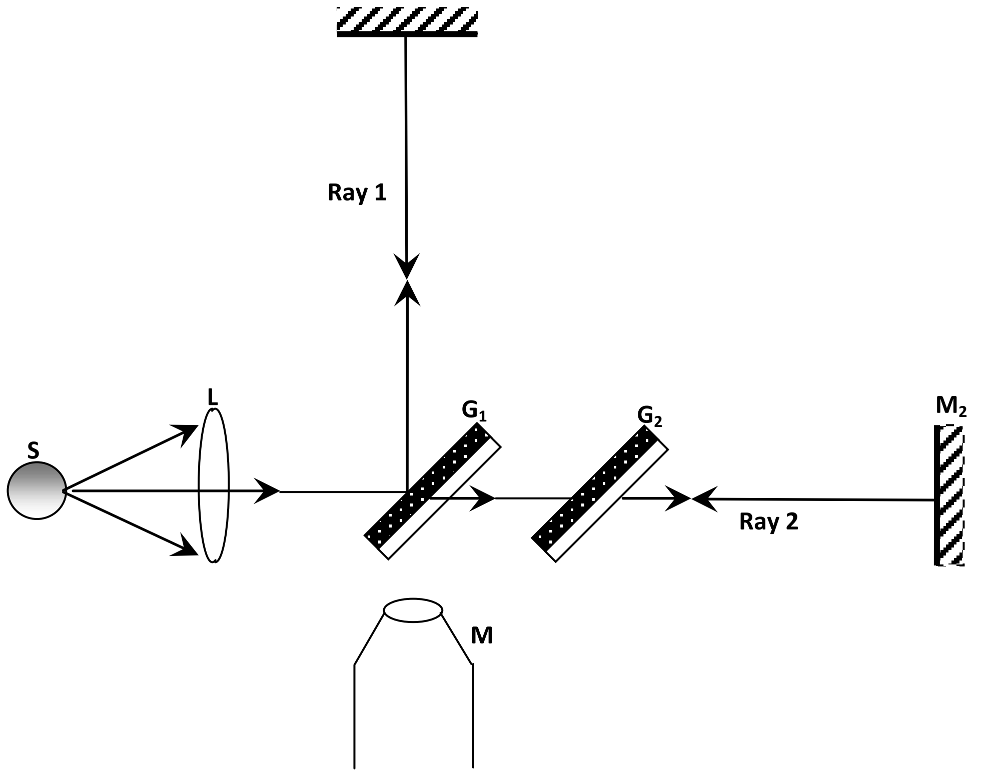
$$\boldsymbol{\lambda = \frac{D_m^2 - D_n^2}{4(m-n)R}} \longrightarrow (1)$$

The diameters of different rings are measured with the help of a travelling microscope. A graph is drawn between the number of the rings and the square of their diameters. The graph is a straight line passing through the origin as shown in figure.



The value of $\frac{D_m^2 - D_n^2}{4(m-n)R}$ is obtained from the graph. Radius of curvature of the Plano convex lens is measured with a spherometer. The values of $\frac{D_m^2 - D_n^2}{4(m-n)R}$ and **R** are substituted in equation 1 to obtain the wavelength $\boldsymbol{\lambda}$ of monochromatic light.

Michelson's Interferometer



Construction:

Michelson's interferometer is shown in figure.

- ✓ S is a monochromatic source of light. L is a convex lens.
- ✓ M_1, M_2 are two plane mirrors which are perpendicular to each other.
- ✓ The two mirrors can be tilted about horizontal and vertical axes with the help of three screws behind the mirror.
- ✓ The mirror M_1 can be moved back and forth using a screw.
- ✓ G_1, G_2 are two half silvered plane mirrors which are parallel to each other.
- ✓ These two mirrors are made of the same material and have the same thickness.
- ✓ The two mirrors are inclined at an angle of $45°$ with respect to M_1, M_2
- ✓ The interference pattern can be observed through the microscope M.

Working:

The light emitted from the source S is converted in to a parallel beam using a convex lens L. This parallel beam of light is divided in to two parts after falling on a semi silvered glass plate G_1. The reflected light travels towards mirror M_1 and the refracted light travels towards the mirror M_2. The two light rays are reflected back from the mirrors M_1, M_2 and meet at G_1 to produce interference. Any desired path difference can be produced between the two rays by moving the mirror M_1 back and forth.

The reflected light ray 1 travels through the glass plate G_1 twice, while the refracted ray 2 does not pass through it even once. Hence the paths of two light rays are not equal. A second glass plate G_2 is introduced in the path of ray 2 between G_1 , M_2 to equalise the paths of the two rays. Hence G_2 is called compensating glass plate.

Let the virtual image of M_2 is formed at M_2'. One of the interfering rays is reflected from mirror M_1 and the other appears to be reflected from the virtual mirror M_2'.

$$OM_1 = x_1, \quad OM_2' = x_2$$

Path difference between the two light rays Δ= $2(x_1 \sim x_2) + \lambda/2$

Determination of wavelength of light using a Michelson interferometer

Michelson's interferometer can be used to determine the wavelength of a monochromatic light. First of all, the two mirrors M_1, M_2 are adjusted for circular rings. Let t be the thickness of the air film.

Condition for bright ring

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

For air film μ=1 and for Normal incidence r=0.

$$2t + \frac{\lambda}{2} = n\lambda$$

If the mirror is moved by a distance of $\frac{\lambda}{2}$ then the next bright spot appears in the field of view. Let **N** be the number of bright fringes that moved across the field of view when the mirror is moved from a position x_1 to a position x_2 .

$$N\,\frac{\lambda}{2} = x_2 - x_1$$

$$\therefore \boldsymbol{\lambda} = \frac{\mathbf{2(x_2 - x_1)}}{\boldsymbol{N}}$$

Determination of difference in wavelengths of sodium D_1, D_2 lines:

First of all, the two mirrors M_1, M_2 are adjusted for circular rings. Let λ_1, λ_2 be the wavelengths of sodium D_1, D_2 lines. The two wavelengths form separate fringe patterns. Since these wavelengths are very close to each other, the patterns approximately coincide. If the mirror M_1 is moved slowly, then the two patterns are separated and the interference pattern becomes blurred. This is because the dark fringe of λ_1 falls on the bright fringe of λ_2. The pattern becomes blurred again when the mirror is moved further by a distance of x.

If x is the displacement of the mirror M_1 between two blurred positions of the pattern, then

$$x = n\frac{\lambda_1}{2} \text{ మరియు } x = (n+1)\frac{\lambda_2}{2}$$

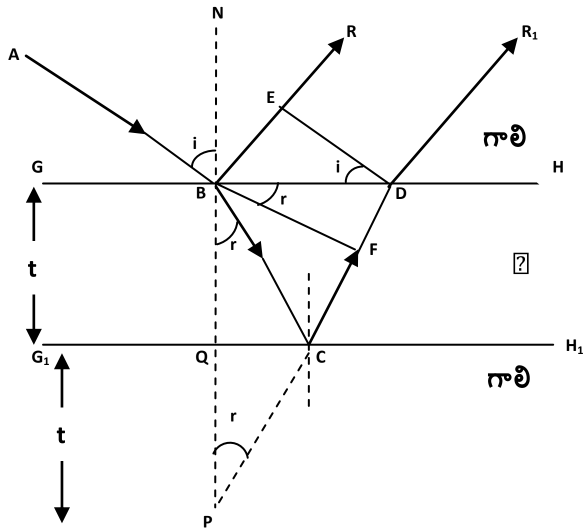
$$n = \frac{2x}{\lambda_1} \text{ మరియు } (n+1) = \frac{2x}{\lambda_2}$$

$$(n+1) - n = \frac{2x}{\lambda_2} - \frac{2x}{\lambda_1} = \frac{2x(\lambda_1 - \lambda_2)}{\lambda_1\lambda_2}$$

$$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2x} = \frac{\lambda_{av}^2}{2x}$$

Cosine Law

Consider a transparent film GHG_1H_1 of thickness t and refractive index μ . Let a monochromatic light of wavelength λ is incident on the film. A part of this light is reflected along **BR** and the remaining part is refracted along BC. The refracted light is reflected back along CD at the point C. It undergoes refraction at the point D and comes out along DR_1 parallel to BR.



$$\angle ABN = \angle BDE = i, \qquad \angle QBC = \angle QPC = R$$

$$\Delta = \mu(BC + CD) - BE$$

$$\mu = \frac{\sin i}{\sin r} = \frac{BE/_{BD}}{FD/_{BD}} = \frac{BE}{FD}$$

$$\therefore BE = \mu(FD)$$

$$\therefore \Delta = \mu(BC + CD) - \mu(FD)$$

$$\Delta = \mu(BC + CF + FD) - \mu(FD)$$

$$\Delta = \mu(BC + CF) = \mu(PF)$$

$$\Delta BPF, \qquad \cos r = \frac{PF}{BP}$$

$$\therefore PF = BP\ \cos r = 2t\ \cos r$$

$$\Delta = \mu \times 2t \cos r = 2\mu t \cos r$$

$$\Delta = \mathbf{2\mu t \cos r}$$

There will be an additional path difference of $\frac{\lambda}{2}$ for light ray **BR** as it undergoes reflection at a denser medium..

$$\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}$$

$$2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$$

$$\mathbf{2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}}$$

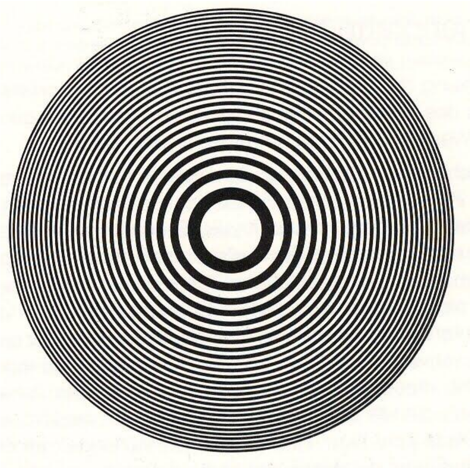
$$2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

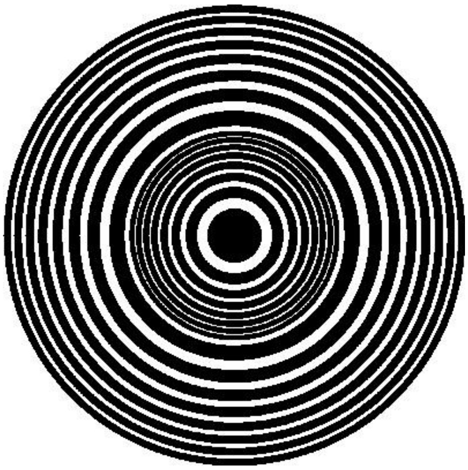
$$\mathbf{2\mu t \cos r = n\lambda}$$

ZONE PLATE

The construction of Fresnel's half period zones can be verified by using an optical device called Zone plate.

Construction of Zone Plate:





The radius of Fresnel's nth half period zone is given by

$$r_n = \sqrt{np\lambda} \propto \sqrt{n}$$

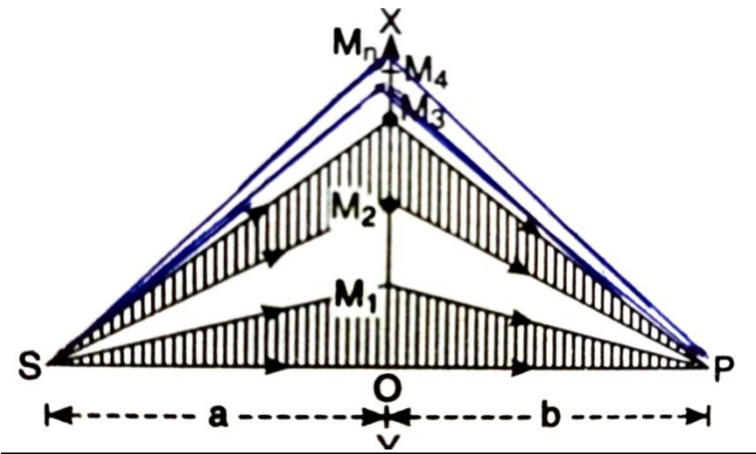
Hence the radii of these zones are proportional to the square roots of natural numbers.

A large number of concentric circles are drawn on a piece of white drawing paper with radii proportional to the square roots of natural numbers. The alternate zones are painted black. Now a reduced size photograph of this drawing is taken. The negative of this photograph is the zone plate. The zones which are painted black on the drawing paper will be transparent in the zone plate.

Zone plates are of two types. Positive zone plate and negative zone plate.

- ✓ A zone plate in which the odd numbered zones are transparent and the even numbered zones are opaque is called a positive zone plate.
- ✓ A zone plate in which the even numbered zones are transparent and odd numbered zones are opaque is called a negative zone plate.

Working of a Zone plate:



The cross section of a zone plate is shown in figure. S is the source of light and P is a point on the screen.

Let $OM_1, OM_2, OM_3 \ldots \ldots$ be the radii of half period zones.
Since there is a path difference of $\lambda/2$ between any two successive zones, from the figure.

$$SM_1 + M_1P = SO + OP + \frac{\lambda}{2} = a + b + \frac{\lambda}{2}$$
$$SM_2 + M_2P = SO + OP + 2\frac{\lambda}{2} = a + b + 2\frac{\lambda}{2}$$

....

....

$$SM_n + M_nP = SO + OP + n\frac{\lambda}{2} = a + b + n\frac{\lambda}{2}$$

From figure

$$(SM_n)^2 = (SO)^2 + (OM_n)^2$$
$$(SM_n)^2 = a^2 + r_n^2$$
$$SM_n = [a^2 + r_n^2]^{1/2}$$
$$= a\left[1 + \frac{r_n^2}{a^2}\right]^{\frac{1}{2}}$$
$$= a\left[1 + \frac{r_n^2}{2a^2}\right]$$
$$SM_n = a + \frac{r_n^2}{2a}$$

Similarly

$$M_nP = b + \frac{r_n^2}{2b}$$
$$\left(a + \frac{r_n^2}{2a}\right) + \left(b + \frac{r_n^2}{2b}\right) = a + b + n\frac{\lambda}{2}$$
$$r_n^2\left(\frac{1}{a} + \frac{1}{b}\right) = n\lambda$$

According to sign convention

$$r_n^2\left(-\frac{1}{a} + \frac{1}{b}\right) = n\lambda$$
$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n}$$

Here $f_n = \frac{r_n^2}{n\lambda}$

The above equation is similar to $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

Hence the focal length of a Zone plate is given by $f_n = \frac{r_n^2}{n\lambda}$

Hence the Zone plate behaves like a convex lens.

Similarities between zone plate and convex lens

S.NO	Zone Plate	Convex Lens
1	$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n}$	$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$
	Both the Zone plate and the convex lens from a real image of the object.	

2	$f_n = \frac{r_n^2}{n\lambda}$	$\frac{1}{f} = (\square - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ $f \propto \frac{1}{(\square - 1)} \propto \lambda$
	The focal lengths of both the Zone plate and the convex lens depend on the wavelength. Hence both exhibit chromatic aberration.	

Differences between Zone plate and convex lens

S.NO	Zone plate	Convex Lens
1	Image is formed due to diffraction.	Image is formed due to refraction
2	A zone plate has multiple focal lengths $f_n = \frac{r_n^2}{n\lambda}$	A convex lens has only one focal length. $\frac{1}{f} = (\square - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
3	In a Zone plate the focal length for red colour is less than the focal length for violet colour. $f_r < f_v$	In a Convex lens, the focal length for red colour is more than the focal length for violet colour. $f_v < f_r$
4	The image formed by a zone plate is less intense.	The image formed by a convex lens is more intense.

Determination of wavelength of given source of light using a diffraction grating:

The wavelength of given source of light can be determined using a diffraction grating.

The condition for principal maximum in diffraction grating is given by

$$(e + d)\sin\theta = n\lambda \longrightarrow (1)$$

Here $e + d$ is the grating element, n is the order of diffraction, θ is the angle of diffraction.

The wavelength λ can be determined by measuring the value of angle of diffraction θ using a spectrometer.

$$(e + d) = 2.54/N$$

Here N is the number of lines on the grating per inch.

Grating Normal Incidence adjustments:

- ✓ The basic adjustments of spectrometer are completed.
- ✓ The spectrometer placed before the sodium vapour lamp and the slit is viewed through the telescope. The telescope is adjusted such that the image of the slit coincides with vertical cross wire of the eye-piece.

- ✓ Now the telescope is rotated to the right of the direct position by an angle of 90 and fixed.
- ✓ Now the prism table is rotated in clock-wise direction. The reflected image of the slit is seen through the telescope and adjusted such that it coincides with vertical cross wire of the eye-piece.
- ✓ Then the prism table is rotated in anti-clockwise direction by an angle of 45 degrees and fixed.
- ✓ Finally the telescope is rotated by an angle of 90 degrees towards the direct reading position.

<u>Measurement of angle of Diffraction:</u>

- ➢ The light emitted from the sodium vapour lamp is diffracted by the gating and forms a spectrum. The spectrum is observed through the telescope.
- ➢ The telescope is turned to the turned to the right and the first order spectrum is observed.
- ➢ The spectral line whose wavelength is to be determined is made to coincide with the vertical cross wire of the eye piece. The reading of the telescope T_1 in this position is taken.
- ➢ Now the telescope is turned to the left and the first order spectrum is observed.
- ➢ The same spectral line is made to coincide with the vertical cross wire of the eye piece. Reading of the telescope T_2 is taken.

Angle of diffraction $\theta = \frac{T_1 - T_2}{2}$

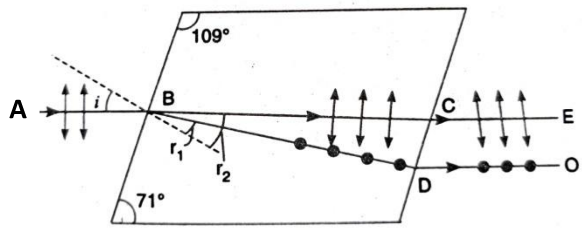
Using this value of θ , the wavelength λ can obtained from equation (1)

Double Refraction

When un-polarised light is passed through a Tourmaline crystal, the refracted light is split in to two refracted rays. This is known as double refraction. The light ray which obeys the ordinary laws of refraction is known as ordinary ray and one which does not obey the laws of refraction is known as extra-ordinary ray. Crystals which exhibit this property is known as doubly refracting crystals.

Ex: Quartz, Mica, Calcite, Tourmaline etc..

Double Refraction is very useful in producing plane polarised light since both the ordinary and extra-ordinary rays are plane polarised.



Let a beam of ordinary light is incident on the calcite crystal with an angle of incidence 'i' as shown in figure. The refracted light is split in to ordinary ray BD and extra-ordinary ray BC. Let r_1, r_2 be the angles of refraction of ordinary and extra-ordinary rays.

Refractive index of ordinary ray $\mu_0 = \frac{sini}{Sin r_1}$

Refractive index of extra-ordinary ray $\mu_e = \frac{sini}{Sin r_2}$

For calcite crystal $\mu_0 > \mu_e$. Hence $v_e > v_0$.

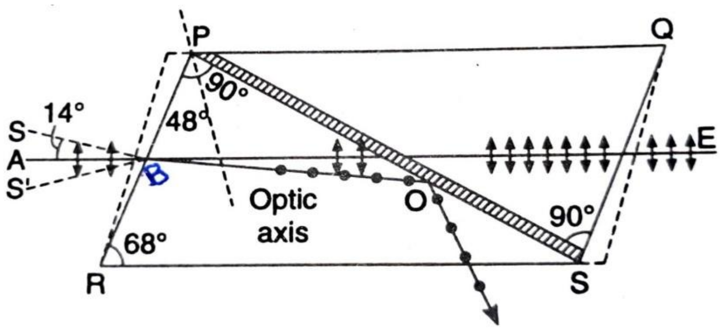
Hence extra-ordinary ray travels faster than ordinary in calcite crystal.

Nicol Prism

Nicol Prism is a device to produce and analyze plane polarized light.

Principle: The working of Nicol's prism is based on the phenomenon of double refraction. When ordinary light is incident on a calcite crystal, the refracted light is split in to ordinary and extra-ordinary rays. Since both these rays are plane polarized, one of these two rays can be eliminated to produce plane polarized light.

Construction:



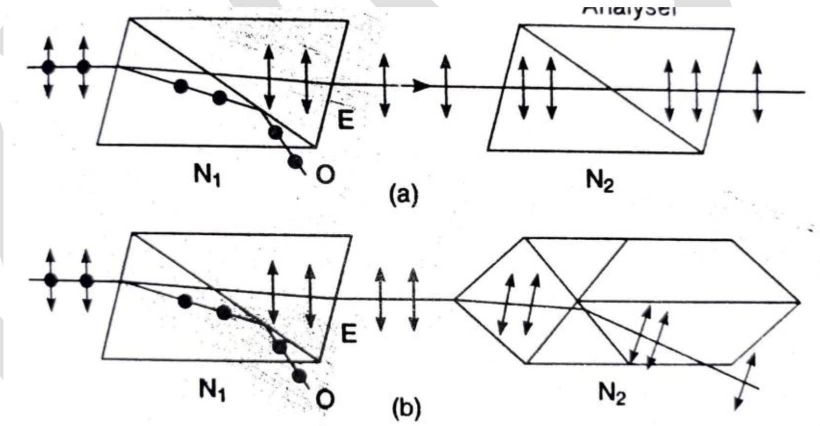
A calcite crystal whose length is three times its width is taken. The end faces of the crystal are grounded such that the angles in the principal section become 68^o, 112^o instead of 71^o, 109^o to increase the field of view. The crystal is cut in to two pieces by a plane AD perpendicular to the principal section. The two pieces are again polished and joined together by a transparent material called Canada balsam. The refractive index of Canada balsam lies in between the refractive indices of ordinary and extra-ordinary rays.

- Refractive index for ordinary rays = 1.658
- Refractive index for Canada balsam =1.55
- Refractive index for extra-ordinary ray=1.486

Working:

Let a beam of un-polarised light is incident on the crystal face PR as shown in figure. The refracted light is split in to ordinary ray BO and extra-ordinary ray BE due to double refraction. Canada balsam acts as a rarer medium for ordinary ray and denser medium for extra-ordinary ray. Since the angles in the principal section are changed to 68^o, 112^o by polishing, the angle of incidence of ordinary ray at the Canada balsam is always greater than the critical angle 69^o. Hence the ordinary ray undergoes total internal reflection at Canada balsam and will be absorbed by the Nicol prism. But the extra-ordinary ray is travelling from rarer medium to denser medium. Hence it will not undergo total internal reflection and will come out of the prism parallel to the incident direction. Since the extra-ordinary ray is plane polarized, the light coming from the Nicol prism is also plane polarised.

Nicol Prism as polariser and analyser

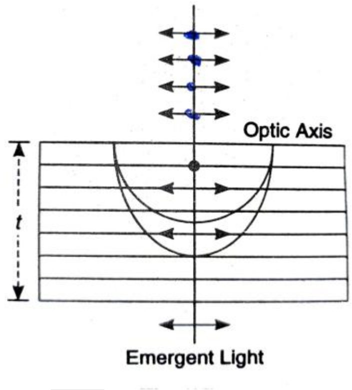


Let two Nicols N_1,N_2 are placed co-axially as shown in figure. The first Nicol N_1 which produces polarised light is known as polariser and the second Nicol N_2 which analyses it is known as analyser. When the principals sections of the two Nicols are parallel to each other, then the polarised light transmitted by N_1 is freely passed through N_2. If the second Nicol N_2 is rotated such that their principal sections are perpendicular to each other then they are in a crossed position. In this position no light comes from the analyser. This is because the extra-ordinary ray becomes ordinary ray after entering the second Nicol and hence undergoes total internal reflection. In this way Nicol prism can be used as both polariser and analyser.

Quarter wave plate

We know that when un-polarised light is incident on calcite crystal, the refracted light splits in to ordinary and extra-ordinary rays. Since the ordinary and extra-ordinary rays travel with different velocities, a path difference is developed between them. The value of path difference depends on the thickness of the crystal.

If the thickness of a crystal is such that it produces a path difference of λ/4 or a phase difference of π/2, then such crystal is known as a Quarter wave plate.



Consider a calcite crystal of thickness 't' as shown in figure. Let μ_0 and μ_e be the refractive indices for ordinary and extra-ordinary rays, then

The path difference between ordinary and extra-ordinary rays= $(\mu_0-\mu_e)t$

For quarter wave plate $(\mu_0-\mu_e)t= \lambda/4$

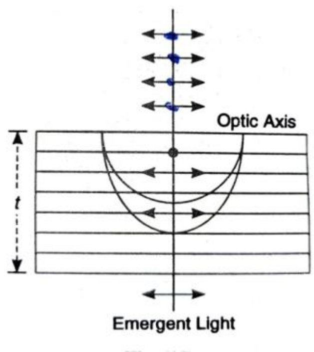
Hence the thickness of a Quarter wave plate $t = \frac{\lambda}{4(\mu_o-\mu_e)}$

It depends on the wavelength of light used.

Half wave plate

We know that when un-polarised light is incident on calcite crystal, the refracted light splits in to ordinary and extra-ordinary rays. Since the ordinary and extra-ordinary rays travel with different velocities, a path difference is developed between them. The value of path difference depends on the thickness of the crystal.

If the thickness of a crystal is such that it produces a path difference of λ/2 or a phase difference of π , then such crystal is known as a Half wave plate.



Consider a calcite crystal of thickness't' as shown in figure. Let μ_o and μ_e be the refractive indices for ordinary and extra-ordinary rays, then

The path difference between ordinary and extra-ordinary rays=$(\mu_0-\mu_e)t$

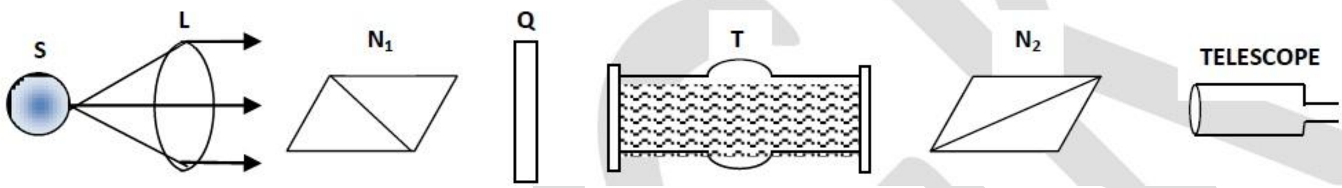
For Half wave plate $(\mu_0-\mu_e)t = \lambda/2$

Hence the thickness of a Half- wave plate $t = \frac{\lambda}{2(\mu_o-\mu_e)}$

It depends on the wavelength of light used.

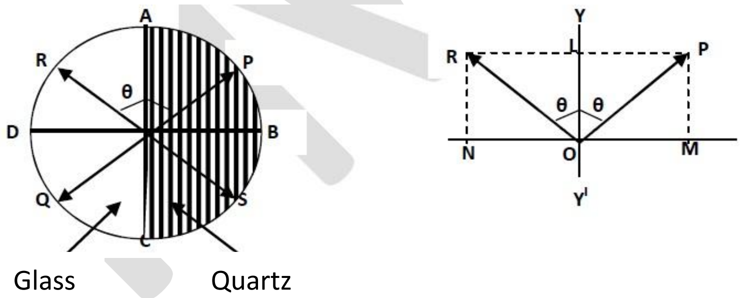
Laurent's Half Shade Polarimeter

When polarized light passes through an optically active substance, its plane of vibration is rotated. The instrument which measures this angle of rotation or specific rotation is known as polarimeter. Laurent's Half Shade Polarimeter is shown in figure.



- S is a monochromatic source of light
- Light emitted from the source S falls on the convex lens L to becomes a parallel beam.
- N_1, N_2 are two Nicol prisms. N_1 acts as polarizer and N_2 acts as analyzer. N_1, N_2 can be rotated about a common axis.
- Q is a half shade device. Polarized light produced by Nicol N_1 passes through the half shade device.
- T is a glass tube. This tube is filled with a solution containing optically active substance.

Working of Half Shade Device



Half shade device contains a semi circular half wave plate ABC and a semi circular glass plate ADC. Let the plane of vibration of plane polarized light incident on the half shade device be along PQ. The polarized light splits into two components inside the quartz plate. Ordinary component is along OM and extra ordinary component is along OL. A phase difference of π is introduced between ordinary and extra ordinary rays due to the half wave plate. Hence the direction of ordinary component is changed from OM to ON. OR is the resultant of ON and OL.

Hence the direction of light from quartz plate is along OP.

- ✓ If the principal axis of the Nicol is parallel to OP, light passing through the glass plate is unobstructed while the light passing through the quartz plate is obstructed. Hence the glass portion is brighter than the quartz portion.
- ✓ If the principal axis of the Nicol is along OR, light passing through the quartz plate is not obstructed while the light passing through the glass portion will be obstructed.
- ✓ If the principal axis of the Nicol is parallel to YY', both the quartz and glass portions will be equally bright. Hence the minimum intensity point can be accurately determined.

Determination of Specific rotation:

Specific rotation is given by

$$S = \frac{\theta}{l \times c}$$

Initially, the glass tube is filled with water and the Nicol N_2 is rotated such that both the glass and quartz halves are equally bright. The two vernier readings V_1, V_2 are noted. Now the water in the glass tube is replaced with the solution for which the specific rotation is to be determined. Now the Nicol N_2 is again rotated such that both the glass and quartz halves are equally bright. Vernier readings are again noted. The difference between the two vernier readings gives the angle of rotation θ.

Experiment is repeated for different known concentrations and the corresponding θ values are noted. A graph is plotted between θ and C and the value of $\frac{\theta}{c}$ is determined from which specific rotation can be calculated.

Unit IV: Aberrations

Chromatic Aberration

Chromatic Aberration:

When a parallel beam of white light is incident on a lens, sometimes the image appears colored and blurred. This aberration is known as chromatic aberration.

- Chromatic aberration is due to light.

Reason for Chromatic Aberration:

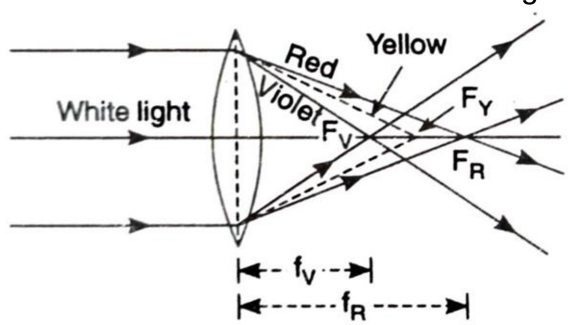
- **Refractive index of the material of a lens changes with the wavelength of light**.

This is the reason for chromatic aberration.

According to lens makers formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \longrightarrow (1)$$

It is clear from the above equation that, focal length of a lens is inversely proportional to its refractive index. Since the refractive index of violet colour is greater than that of red colour, focal length for violet colour is less than that of red colour. As a result violet rays are focussed nearer to the lens and red rays are focussed farther from the lens. Hence the image appears coloured and blurred.



Minimization of Chromatic Aberration- Achromatic Doublet

Chromatic aberration is positive for convex lens and negative for concave lens. Hence a combination of convex and concave lenses can be used to minimize chromatic aberration. This is known as Achromatic Doublet and the phenomenon is known as Achromatism.

Consider a combination of two lenses of focal lengths f_1 and f_2.

According to lens makers formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \longrightarrow (1)$$

Differentiating the above equation on both sides, we get

$$d\left(\frac{1}{f}\right) = d\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \longrightarrow (2)$$

Dividing equation eqn 2 with eqn 1, we get

$$f\, d\left(\frac{1}{f}\right) = \frac{d\mu}{\mu - 1}$$

Here $\frac{d\mu}{\mu-1} = \omega =$ Dispersive power

$$\therefore\ f\, d\left(\frac{1}{f}\right) = \omega$$

$$\boldsymbol{d\left(\frac{1}{f}\right) = \frac{\omega}{f}} \quad \longrightarrow \quad (3)$$

Resultant focal length is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Differentiating the above equation on both sides, we get

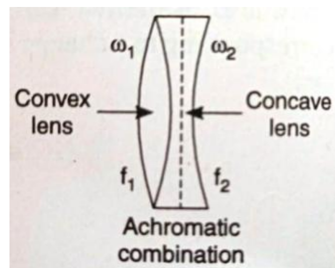
$$d\left(\frac{1}{f}\right) = d\left(\frac{1}{f_1}\right) + d\left(\frac{1}{f_2}\right)$$

From eqn. 3,

$$d\left(\frac{1}{f_1}\right) = \frac{\omega_1}{f_1} \quad and \quad d\left(\frac{1}{f_2}\right) = \frac{\omega_2}{f_2}$$

$$d\left(\frac{1}{f}\right) = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2}$$

To minimize chromatic aberration, focal length should be constant.
Hence



$$d\left(\frac{1}{f}\right) = 0$$

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$$\frac{\omega_1}{f_1} = -\frac{\omega_2}{f_2}$$

$$\boldsymbol{\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}}$$

- **To minimize chromatic aberration, ratio of focal lengths of the two lenses should be equal to the negative ratio of their dispersive powers.**

<u>Minimization of chromatic aberration using two convex lenses separated by a distance</u>

Consider two lenses of focal lengths f_1 and f_2 separated by a distance x.
According to lens makers formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \longrightarrow \quad (1)$$

Differentiating the above equation on both sides, we get

$$d\left(\frac{1}{f}\right) = d\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \longrightarrow \quad (2)$$

Dividing equation eqn 2 with eqn 1, we get

$$f\, d\left(\frac{1}{f}\right) = \frac{d\mu}{\mu - 1}$$

Here $\frac{d\mu}{\mu-1} = \omega =$ Dispersive power

$$\therefore\ f\, d\left(\frac{1}{f}\right) = \omega$$

$$\boldsymbol{d\left(\frac{1}{f}\right) = \frac{\omega}{f}} \quad \longrightarrow \quad (3)$$

Resultant focal length is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

Differentiating the above equation on both sides, we get

$$d\left(\frac{1}{f}\right) = d\left(\frac{1}{f_1}\right) + d\left(\frac{1}{f_2}\right) - x\left[\frac{1}{f_1} d\left(\frac{1}{f_2}\right) + \frac{1}{f_2} d\left(\frac{1}{f_1}\right)\right]$$

From eqn. 3,

$$d\left(\frac{1}{f_1}\right) = \frac{\omega_1}{f_1} \quad and \quad d\left(\frac{1}{f_2}\right) = \frac{\omega_2}{f_2}$$

$$d\left(\frac{1}{f}\right) = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - x\left[\frac{1}{f_1}\frac{\omega_1}{f_1} + \frac{1}{f_2}\frac{\omega_2}{f_2}\right]$$

To minimize chromatic aberration, focal length should be constant.
Hence

$$d\left(\frac{1}{f}\right) = 0$$

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - x\left[\frac{1}{f_1}\frac{\omega_1}{f_1} + \frac{1}{f_2}\frac{\omega_2}{f_2}\right] = 0$$

$$\boldsymbol{x = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2}}$$

If the two lenses are made of the same material, then $\omega_1 = \omega_2$

$$\boldsymbol{x = \frac{f_1 + f_2}{2}}$$

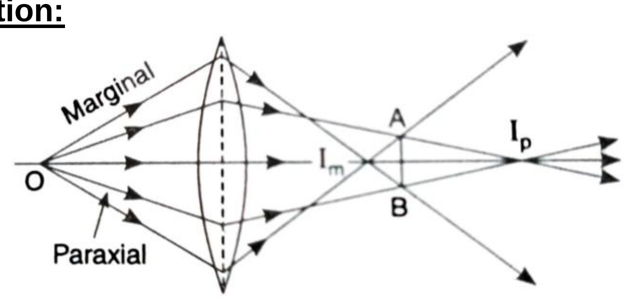
- **To minimize chromatic aberration, separation between two lenses should be equal to the mean of their focal lengths**

<u>Spherical Aberration</u>

If the image of a point object placed on the principal axis of a lens is blurred, such an aberration is known as Spherical Aberration.

- Spherical aberration is due to the spherical surface of the lens.

<u>Reason for Spherical Aberration:</u>



When a beam of parallel light rays are incident on the lens, the deviation of light rays is directly proportional to the height.

$$\delta = \frac{h}{f}$$

Hence light rays which are incident nearer to the principal axis have small deviation and are focused farther from the lens at the point I_p. These rays are called paraxial rays. Similarly light rays which are incident farther from the axis have large deviation and are focused nearer to the lens at the point I_m.

These rays are called marginal rays. Hence focal length for paraxial rays is greater than that of marginal rays. Hence the images appears blurred.

Minimization of Spherical Aberration:

1. **Using plano-convex lenses:**

   Spherical aberration can be minimized using plano-convex lenses. Consider a plano-convex LENS as shown in figure. Let δ_1, δ_2 be the deviations of the light ray at the two surfaces

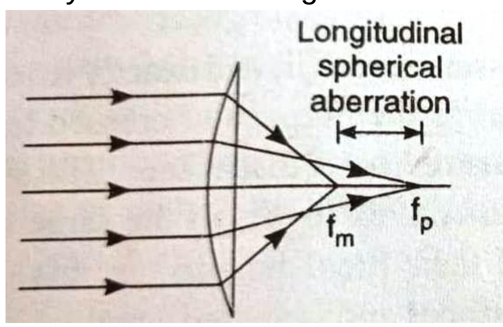
$$\text{Total deviation } \delta = \delta_1 + \delta_2$$

   Spherical aberration is directly proportional to the square of minimum deviation.

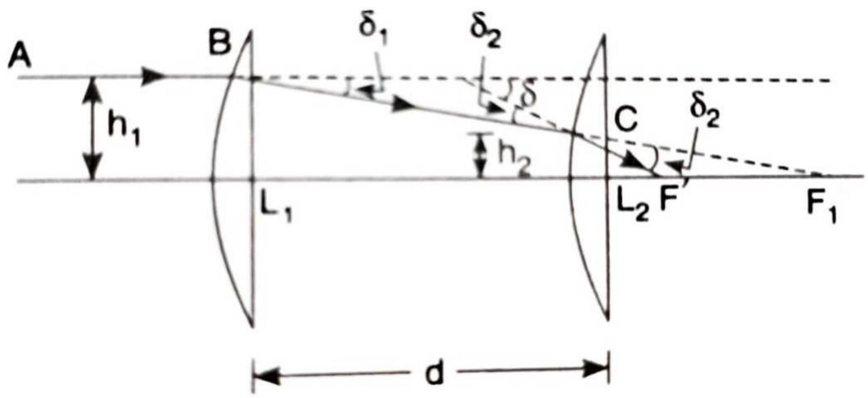
$$\text{Spherical aberration} \propto \delta^2$$
$$\propto (\delta_1 + \delta_2)^2$$
$$\propto (\delta_1 - \delta_2)^2 + 4\delta_1 \delta_2$$

   It is clear from the above equation that spherical aberration is minimum when $\delta_1 = \delta_2$

   Hence to minimize spherical aberration using a plano-convex lens, the convex side should face the incident beam of parallel light rays as shown in figure.



2. **Using two plano-convex lenses separated by a distance:**



   Spherical aberration can be minimized using two plano-convex lenses separated by a distance as shown in figure.

   Consider two plano-convex lenses L_1 and L_2 of focal lengths f_1 and f_2 separated by a distance d as shown in figure. Consider a light ray AB incident on the lens L_1 at a height h_1. The refracted light ray BC strikes the lens L_2 at a height h_2.

   Condition for minimization of spherical aberration

$$\delta_1 = \delta_2$$
$$\frac{h_1}{f_1} = \frac{h_2}{f_2}$$

$$\frac{h_1}{h_2} = \frac{f_1}{f_2} \longrightarrow (1)$$

From similar triangles, BL_1F_1 and CL_2F_1

$$\frac{h_1}{h_2} = \frac{L_1F_1}{L_2F_1} = \frac{f_1}{f_1 - d} \longrightarrow (2)$$

From equations 1 and 2

$$\frac{h_1}{h_2} = \frac{f_1}{f_2} = \frac{f_1}{f_1 - d}$$

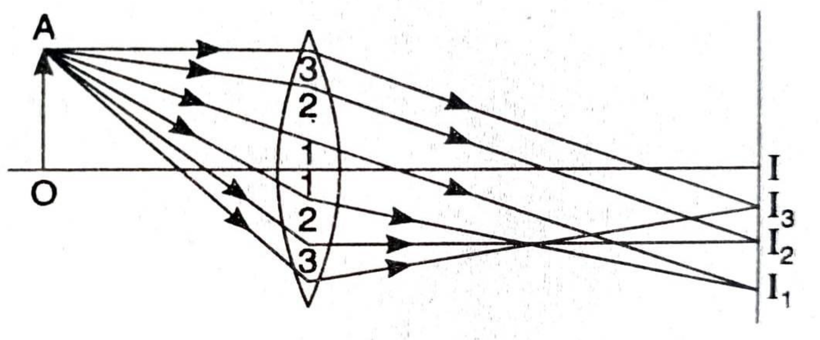
$$f_2 = f_1 - d$$

$$\boldsymbol{d = f_1 - f_2}$$

- **To minimize spherical aberration, separation between the two lenses should be equal to the difference of their focal lengths.**

COMA

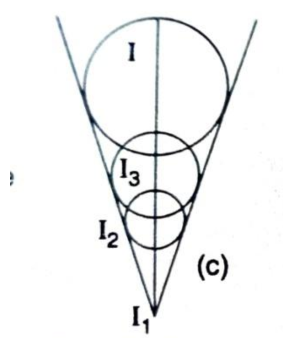
When a point object is placed slightly away from the axis, the image of the point object is formed in the shape of a comet. This aberration is known as coma.



Reason for coma:

1. Different zones of the lens produce different lateral magnifications.
2. Each zone forms the image in the form of a circle.

Consider a point object placed slightly away from the axis at the point A as shown in figure. Light rays passing through the zones $(1,1)$ $(2,2)$ $(3,3)$ …. are focused at different points I_1, I_2, I_3 ….. . Hence the image appears in the shape of a comet.



Resultant image due to different zones

Elimination of coma:

1. Coma can be minimized by using stops by restricting the outer zones.
2. Lenses satisfying the abbe sine condition do not exhibit coma.
   Abbe sine condition is given by

$$\mu_1 y_1 \sin\theta_1 = \mu_2 y_2 \sin\theta_2$$

$\theta_1, \theta_2 =$ Angles of the rays with the axis

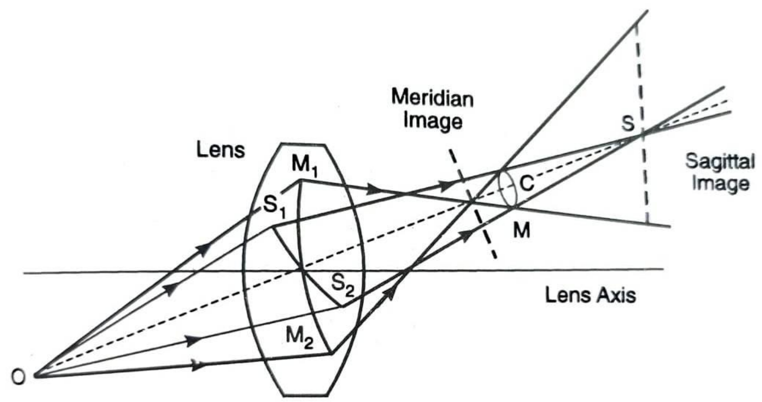
$y_1, y_2 =$ Lengths of object and image

$\mu_1, \mu_2 =$ Refractive indices of object and image spaces

ASTIGMATISM

When a point object is places far away from the axis, the image formed by the lens consists of two mutually perpendicular lines separated by a distance. This aberration is known as astigmatism.

Consider a point object O placed far away from the axis as shown in figure.



- Plane containing the object O and the principal axis of the lens is called meridian plane M_1M_2
- Plane perpendicular to the meridian plane and passing through the principal axis is called sagittal plane S_1S_2

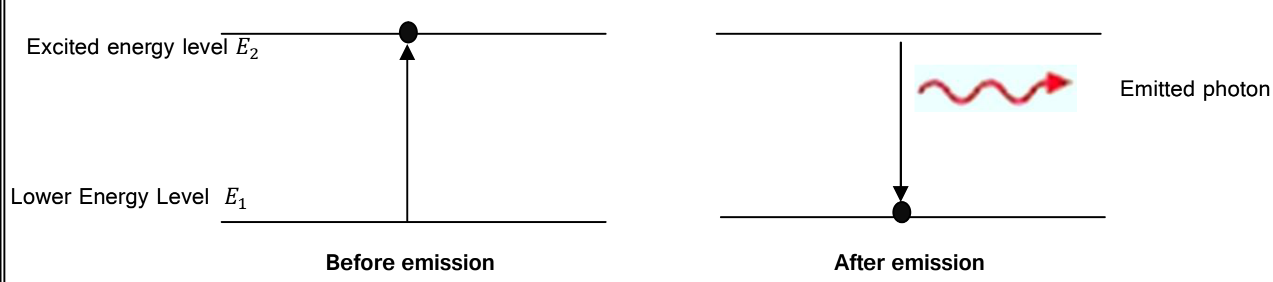
Light rays passing through the meridian plane are focused on a horizontal line M while the light rays passing through the sagittal plane are focused on a vertical line S. Hence the image consists of two mutually perpendicular lines separated by a distance. When the screen is moved between M and S, a circle is formed. This is known as circle of least confusion.

Elimination of astigmatism:

1. Astigmatism can be minimized by using stops by restricting the outer zones.
2. Astigmatism is positive for convex lens and negative for concave lens. Hence a combination of convex and concave lenses can be used to minimize astigmatism.

LASER

SPONTANEOUS EMISSION



Consider an atom excited to a higher energy state E_2 from a lower energy state E_1 as shown in figure. The atom cannot remain in this excited state for more than $10^{-8}\,Sec$. If the atom de excites to the lower energy state by the emission of a photon of energy $hv = E_2 - E_1$ without any external influence, then such emission is called spontaneous emission.

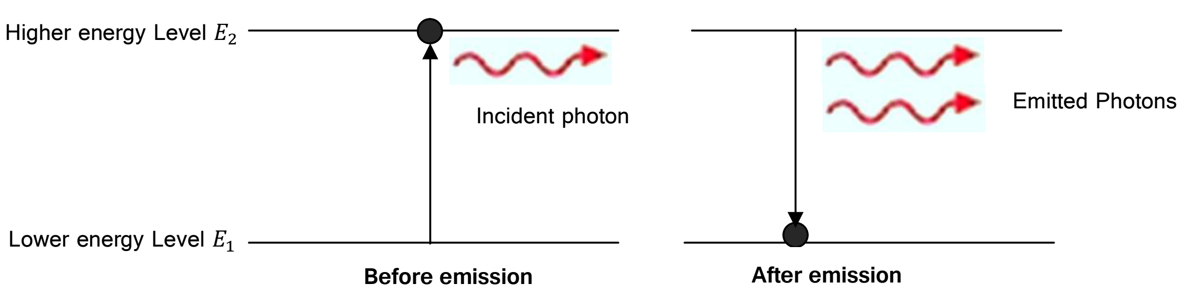
- ✓ The rate of spontaneous emission depends only on the number of atoms N_2 in the higher energy state E_2. It does not depend on the intensity of incident radiation $\rho(v)$.

Let N_{sp} be the number of spontaneous emission transitions per unit time.Let N_2 be the number of atoms in the higher energy state

$$\therefore\ N_{sp} \propto N_2$$
$$N_{sp} = A_{21}N_2$$

Here A_{21} is called Einstein's coefficient of spontaneous emission.

STIMULATED EMISSION



Consider an atom excited to a higher energy state E_2 from a lower energy state E_1 as shown in figure. The atom cannot remain in this excited state for not more than $10^{-8}\,Sec$. If a photon of energy $hv = E_2 - E_1$ hits this atom, then the atom de excites to the lower energy state by the emission of a photon of the same frequency. Hence two identical photons are emitted in this process.

- ✓ The rate of stimulated emission depends on the number of atoms in the excited state N_2 and the intensity of incident radiation $\rho(v)$.

Let N_{st} be the number of stimulated emission transitions per unit time.Let N_2 be the number of atoms in the higher energy state.

$$\therefore\ N_{st} \propto N_2\,\rho(v)$$
$$N_{st} = B_{21}N_2\,\rho(v)$$

Here B_{21} is called Einstein's coefficient of stimulated emission.

Population Inversion

Generally, the number of atoms in an excited energy level is less than the number of atoms in the ground state. But the number of atoms in the excited state must be greater than the number of atoms in the ground state to produce Laser light. This is called population inversion.

- ✓ The process of achieving population inversion is called Pumping.
- ✓ The material in which population inversion takes place is called the active material.

If N_1 is the number of atoms in the ground state E_1 and N_2 is the number of atoms in the excited energy level E_2, then according to Maxwell-Boltzman distribution law

$$\frac{N_1}{N_2} = e^{\frac{(E_2-E_1)}{kT}}$$

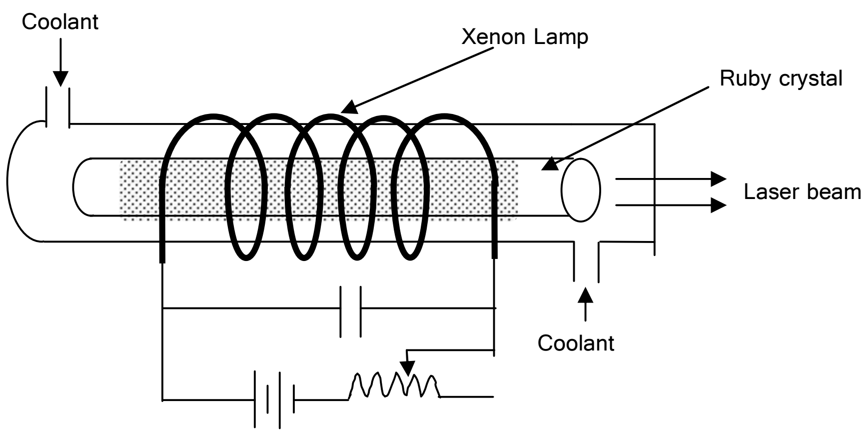
Generally $N_1 > N_2$.

RUBY LASER

Ruby crystal is formed by adding chromium atoms to Aluminium oxide crystal Al_2O_3. The Chromium ions (Cr^{+3}) act as the active material in the Ruby crystal.

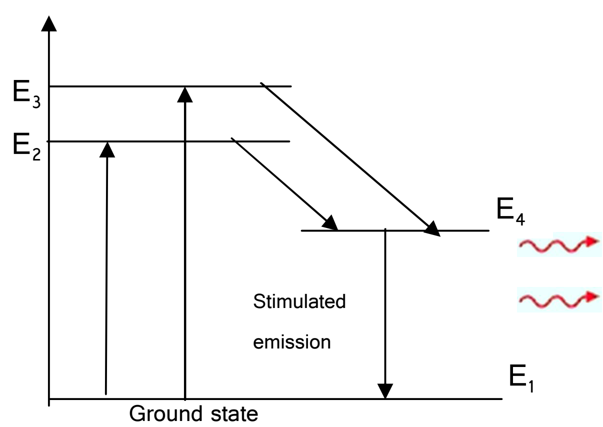
Construction of Ruby Laser is shown in figure.

- Ruby crystal is taken in the form of a cylindrical rod of length 10cm and diameter 1cm .The two ends of the cylinder are polished such that they are perpendicular to the axis of the cylinder.
- One end of the rod is completely silvered fully reflect light.
- Other end of the rod is partially silvered to be semi transparent.
- Ruby rod is surrounded by a helical Xenon lamp.
- The rod is also surrounded by a cooling agent like liquid nitrogen.



Working:

Ruby laser is a three level laser. The chromium ions act as the active material in Ruby laser. The energy levels of chromium ions are shown in figure.



- The chromium ions in the ground state E_1 will be excited to the higher energy states E_2 and E_3 by absorbing the light emitted from the Xenon lamp.
- But the atoms cannot remain in these excited energy states E_2 and E_3 for not more than $10^{-8}\,Sec$. Hence they de excite to a lower energy level E_4 by radiationless transition.
- Since the energy state E_4 is a Meta stable state, the atoms remain in this energy state for a relatively long time.
- Hence the population of energy state E_4 becomes more than the population of ground state E_1. As a result population inversion is achieved.
- The atoms in E_4 energy state emit photons of wavelength 6943 A^0 through spontaneous emission.
- The emitted photon travels back and forth along the axis of the rod and stimulates the emission of another photon of same wavelength and same phase. This is called stimulated emission.

❖ This process is repeated again and again until an intense laser beam is produced which emerges parallel to the axis of the rod.

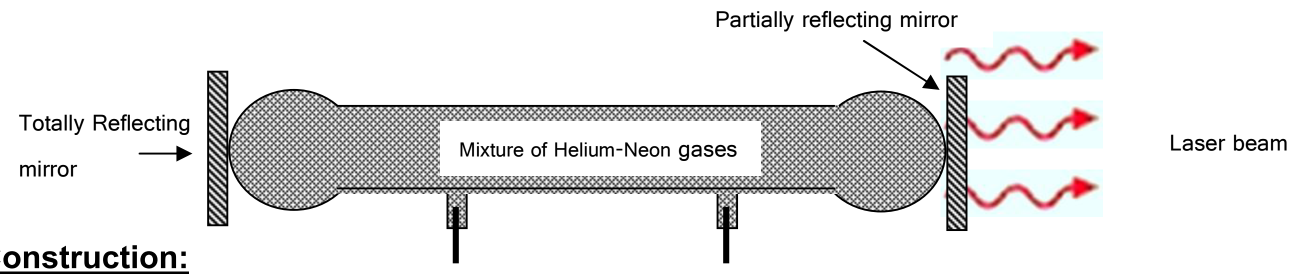
Drwabacks:

- The laser output is not continuous.
- The active material absorbs only the green component of light emitted from the xenon lamp. Hence the efficiency of Ruby laser is low.

Helium-Neon Laser

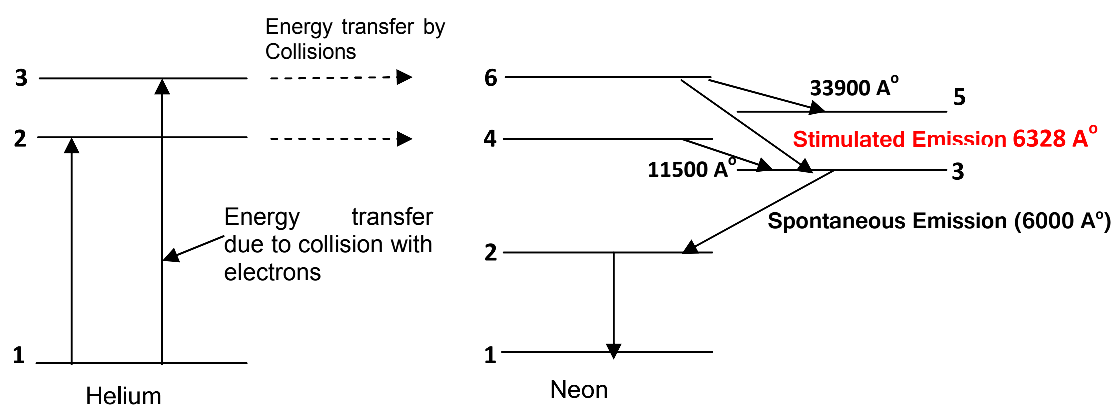
Helium-Neon laser is the first gas laser. In Helium-Neon Laser, a mixture of Helium-Neon Gas acts as the active material.

Helium-Neon Laser is shown in figure.



Construction:

- It contains a discharge tube of length 50cm and diameter 1cm.
- The discharge tube contains a mixture of Helium and Neon gases in the ratio of 10:1
- The discharge tube also contains electrodes to produce electric discharge. These electrodes are connected to a high voltage battery.
- One end of the rod is completely silvered to fully reflect light.
- Other end of the rod is partially silvered to be semi transparent.



Working:

The energy levels of Helium and Neon are shown in figure. Laser light is produced due to the transitions between the energy levels of Neon atoms. Helium atoms only help in pumping the neon atoms to higher energy levels.

❖ The electrons produced in the discharge tube excite the Helium atoms in to meta stable states He_2, He_3 .

❖ The Helium atoms in these Meta stable states He_2, He_3 return to ground sate by transferring their energy to neon atoms.

❖ As a result, the Neon atoms are excited to the meta stable states Ne_4, Ne_6 .

❖ The Helium atoms in the ground state are repeatedly excited by electric discharge and there by continuously pump neon atoms to the Meta stable states.

❖ Since Helium and neon gases are in the ratio of 10:1, the probability for transfer of energy from Neon atoms to Helium atoms is very less.

❖ Hence the population of neon atoms in energy states Ne_4, Ne_6 increases causing population inversion.

- ❖ Ne_3, Ne_5 Serve as lower energy levels for laser action.

Three different transitions are possible.

$Ne_6 \longrightarrow Ne_3$: Red color laser light of wavelength 6328 is produced due to this transition.

$Ne_6 \longrightarrow Ne_5$: Laser light of wavelength 33900 A^0 is produced due to this transition.

$Ne_4 \longrightarrow Ne_3$: Laser light of wavelength 11500 A^0 is produced due to this transition.

The two ends of the discharge tube are designed such that laser light of wavelengths 33900 A^0 , 11500 A^0 is absorbed. Hence only laser light of wavelength 6328 A^0 comes out of the tube.

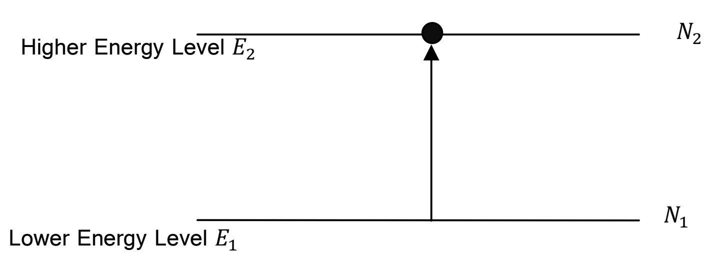
Applications:

- ✓ Helium-neon laser is used in interferometers.
- ✓ Helium-neon laser is used in meteorology.
- ✓ Helium-Neon laser is used in bar-code scanners.

Einstein's Coefficients

Let N_1 be the number of atoms in the lower energy level $E_1$and N_2 be the number of atoms E_2 in the excited energy level.

When a photon of frequency $\upsilon = \frac{E_2-E_1}{h}$ is incident on the system, three processes can take place.



1. **Stimulated Absorption:**

If an atom in the ground state E_1 is excited to a higher enrgy state E_2 by absorbing the incident light, the process is called stimulated absorption.

- ➢ The rate of stimulated absorption depends on the number of atoms in the ground state E_1 and the energy density of incident radiation $\rho(\upsilon)$.

$$N_{ab} \propto N_1\, \rho(\upsilon)$$

$$\therefore\ N_{ab} = B_{12}\, N_1\, \rho(\upsilon)$$

Here B_{12} is called Einstein's coefficient of stimulated absorption.

2. **Spontaneous Emission:**

Consider an atom excited to a higher energy state E_2 from a lower energy state E_1 . The atom cannot remain in this excited state for not more than $10^{-8}\, Sec$. If the atom de excites to the lower energy state by the emission of a photon of energy $h\upsilon = E_2 - E_1$ without any external influence, then such emission is called spontaneous emission.

- ➢ The rate of spontaneous emission depends only on the number of atoms N_2 in the higher energy state E_2. It does not depend on the intensity of incident radiation $\rho(\upsilon)$.

Let N_{sp} be the number of spontaneous emission transitions per unit time. Let N_2 be the number of atoms in the higher energy state

$$\therefore\ N_{sp} \propto N_2$$

$$N_{sp} = A_{21} N_2$$

Here A_{21} is called Einstein's coefficient of spontaneous emission.

3. **Stimulated Emission:**

Consider an atom excited to a higher energy state E_2 from a lower energy state E_1 as shown in figure. The atom cannot remain in this excited state for not more than $10^{-8}\, Sec$. If a photon of energy

$h\upsilon = E_2 - E_1$ hits this atom, then the atom de excites to the lower energy state by the emission of a photon of the same frequency. Hence two identical photons are emitted in this process.

- The rate of stimulated emission depends on the number of atoms in the excited state N_2 and the intensity of incident radiation $\rho(\upsilon)$.

Let N_{st} be the number of stimulated emission transitions per unit time. Let N_2 be the number of atoms in the higher energy state.

$$\therefore\ N_{st} \propto N_2\, \rho(\upsilon)$$

$$N_{st} = B_{21} N_2\, \rho(\upsilon)$$

Here B_{21} is called Einstein's coefficient of stimulated emission.

$$N_{ab} = N_{sp} + N_{st}$$

$$B_{12}\, N_1\, \rho(\upsilon) = A_{21} N_2 + B_{21} N_2\, \rho(\upsilon)$$

$$(B_{12}\, N_1 - B_{21} N_2)\rho(\upsilon) = A_{21} N_2$$

$$\rho(\upsilon) = \frac{A_{21} N_2}{B_{12}\, N_1 - B_{21} N_2}$$

Dividing numerator and denominator with $B_{21} N_2$

$$\rho(\upsilon) = \frac{\left(\frac{A_{21}}{B_{21}}\right)}{\left(\frac{B_{12} N_1}{B_{21} N_2} - 1\right)}$$

From Boltzman distribution law $N_i = N_0\, E^{-\left(E_i/kT\right)}$

Hence $N_1 = N_0\, E^{-\left(E_1/kT\right)}$

$$N_2 = N_0\, E^{-\left(E_2/kT\right)}$$

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/kT} = e^{h\upsilon/kT}$$

$$\rho(\upsilon) = \frac{\frac{A_{21}}{B_{21}}}{\left[\left(\frac{B_{12}}{B_{21}}\right) e^{h\upsilon/kT} - 1\right]} \longrightarrow (1)$$

According to Planck's law

$$\rho(\upsilon) = \frac{\left(\frac{8\pi h\upsilon^3}{c^3}\right)}{e^{h\upsilon/kT} - 1} \longrightarrow (2)$$

Comparing equations 1 and 2,

$$\boldsymbol{B_{12} = B_{21}}$$

$$\boldsymbol{\frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{c^3}}$$

The above equations are called Einstein's relations.

Hence the ratio of spontaneous emission to stimulated emission is proportional to the cube of the frequency of incident light.

Applications of Lasers

- Lasers are used in measuring distances. This is called Light Detection and Ranging (LIDAR)
- Laser light is used in drilling and cutting of metals.
- Laser light is used in cataract surgery for eyes.
- Laser light is used in the diagnosis and treatment of cancer.
- Laser light is used in achieving Nuclear Fusion.
- Laser light is used in reading and writing digital information in CDs and DVDs.
- Laser light is used in holography.